NOTATION

A, surface area; D, diameter; f, fraction of the surface that is in contact with the bubbles; g, acceleration of gravity; k, optical contrast of the phase; m, porosity; q, radiant flux density; R, radius; T, absolute temperature; t, time, u, velocity; ε, degree of blackness; ϰ, degree of bubble deformation, ν, kinematic viscosity; σ, Stefan–Boltzmann constant; Re, Reynolds number; Ar, Archimedes number. Subscripts: 0, start of fluidization; b, bubble; em, emulsion phase; fb, fluidized bed; g, gas; p, particle; r, radiometer; w, wall.

LITERATURE CITED

1. V. I. Kovenskii, "Calculation of emittance of a disperse system," Inzh.-Fiz. Zh., 38, No. 6, 983-988 (1980).
2. V. A. Borodulya and V. I. Kovenskii, "Calculating radiant heat exchange between a fluidized bed and a surface," Inzh.-Fiz. Zh., 40, No. 3, 466-472 (1981).
3. V. A. Borodulya, V. L. Ganzha, and V. I. Kovenskii, Hydrodynamics and Heat Exchange in a Fluidized Bed Under Pressure [in Russian], Nauka i Tekhnika, Minsk (1982).
4. K. E. Makhorin, V. S. Pikashov, nad G. P. Kuchin, Heat Exchange in a High-Temperature Fluidized Bed [in Russian], Naukova Dumka, Kiev (1981).
5. D. Kunin and O. Levenshpil', Industrial Use of Fluidized Beds [in Russian], Khimiya, Moscow (1976).
6. J. F. Davidson and D. Harrison (eds.), Fluidization, Academic Press (1971).
7. A. E. Sheindlin (ed.), Radiant Properties of Solid Materials (Handbook) [in Russian], Énergiya, Moscow (1974).

RADIATIVE–CONDUCTIVE HEAT TRANSFER IN "HEATER–MULTILAYER STRUCTURE" SYSTEM

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UDC 536.24

The coupled problem of radiative–conductive heat transfer is solved by a numerical method. The integral equation describing radiative heat transfer is approximated with a system of linear algebraic equations.

The main purposes of a thermal experiment are identifying the structure of the mathematical models of the thermal state and analyzing heat-transfer processes which actually occur in objects under study. Natural heating tests are extremely complicated and costly. For this reason, wide acceptance have received testing methods based on studying the thermal state of a natural structure on simulating test stands such as, for instance, test stands simulating radiative heating. Performing such tests requres careful preparation and, above all, scientifically substantiated rational planning. Lately new approaches are taken in many places not only to processing of experimental data but also to optimal organizing of experiments. One way to improve the effectiveness of experimental studies is applying modern methods of mathematical simulation of the thermal state of an object under test stand conditions during the test preparation stage.

In this study will be developed a mathematical model of heating of an axisymmetric structure during tests performed in radiative heating stand, a model based on the solution to the coupled problem of radiative–conductive heat transfer.

The mathematical model of radiative–conductive heat transfer involves a simultaneous solution of two problems, radiative heat transfer in a "heater–irradiated surface" system and transient heat transfer in a multilayer structure.

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External Problem of Radiative Heat Transfer. The distribution of the density of integral thermal radiation flux over the surface of the structure under study is determined through analysis of the radiative heat transfer in the system, the latter containing the irradiated axisymmetric surface of any intricate shape and a centrally located cylindrical radiator. The medium between diffusely emitting surfaces and diffusely reflecting surfaces in the system is assumed to be diathermal, and the emissivity at every point of a surface is assumed to be equal to the absorptivity.

With known geometry, optical characteristics, and temperature at every point of the system surfaces, the process of radiative heat transfer is described by an integral equation which can be written for the effective radiation density as

$$E_{\text{eff}}(M) - R(M)\int_F E_{\text{eff}}(N)K(M,\ N)\,dF = E_{\text{c}}(M),\quad M, N\in F. \tag{1}$$

An analytical solution of Eq. (1) is possible only for a limited number of systems, geometrically rather simple ones. For this reason, a wide acceptance have received approximate methods of solving problems of radiative heat transfer, specifically the zonal method: approximating a surface of the given system with plane zones within which the density of effective radiation fluxes are either averaged according to some law or are assumed to be constant on the basis of certain premises.

In this case the integral equation (1) is replaced with the system of algebraic equations

$$E_{\text{eff}i} - R_i\sum_{k=1}^{N} E_{\text{eff}k}\,\varphi_{ik} = E_{\text{c}_i},\ i = 1,\ 2,\ \dots,\ N, \tag{2}$$

with the direction coefficient of mutual irradiance φ_{ik} generally defined by the expression

$$\varphi_{ik} = \frac{1}{F_i}\int_{F_i}\int_{F_k}\frac{\cos\alpha_i\cos\alpha_k}{\pi r^2}\,dF_k dF_i. \tag{3}$$

The choice of method selected for solving this system of linear algebraic equations is very important, since solution of the problem of coupled radiative-conductive heat transfer requires repeatedly determining the field of thermal radiation fluxes. Methods of solving systems of linear algebraic equations have been analyzed and optimal selection of the initial approximation has been considered in another study [1]. Here the density of thermal fluxes fed to a structure was determined from the solution to the system of linear algebraic equations by the Seidel iteration method.

The main difficulties in studying radiative heat transfer in this formulation by the zonal method arise during determination of the mean direction coefficents φ_{ik} of mutual irradiance. They can be calculated by various methods. Several studies have been published already [2, 3] where the authors deal with determination of φ_{ik} in axisymmetric three-dimensional systems. The capacity of direct-access memories and the high speed of modern computers make it feasible to effectively use numerical methods for determining these direction coefficients. In this study they will be calculated by a numerical method.

Existence of axial symmetry in a given system permits approximating a surface of such a system with annular zones rather than subdividing the system into plane zones. Therefore,

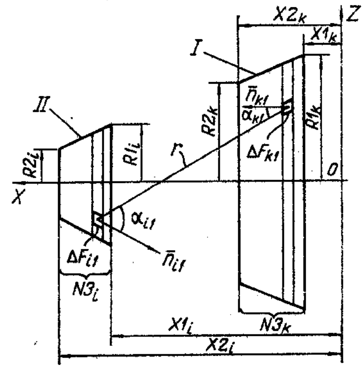

Fig. 1. Determination of direction coefficients in axisymmetric system: I) k-th annular zone; II) i-th annular zone.

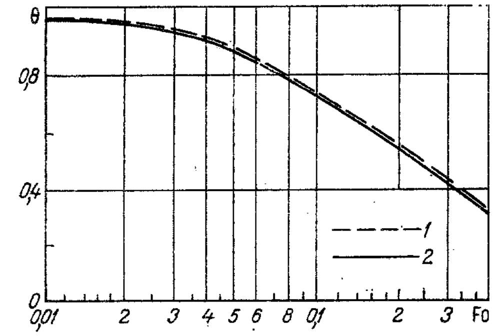

Fig. 2. Comparison of results of our calculations with published data [6] pertaining to the problem with boundary conditions of the I-II kind: 1) results of numerical solution; 2) results of analytical solution [6].

unlike in determination of direction coefficients for solution of problems of radiative heat transfer in the three-dimensional formulation, in the case of axisymmetric three-dimensional systems there is no need to stipulate a rather unwieldy amount of initial data on the system geometry in the form of arrays of coordinates locating the vertices of approximating plane zones, such zones having been replaced here with annular ones (generally in the shape of truncated cones). The initial data on location of the i-th approximating annular zone in space is unambiguously stipulated by two coordinates of their outermost sections $X1_i$ and $X2_i$ along the X axis, sections which have been formed upon subdivision of the initial surface of the axisymmetric structure by planes perpendicular to the X axis, and by the radii $R1_i$ and $R2_i$ of circles in these two sections (Fig. 1).

The direction coefficients are calculated taking into account the plane symmetry of the system with respect to the XOZ plane, while the reciprocality relation between direction coefficients makes it unnecessary to calculate on the computer and store in its direct-access memory more than the half matrix of direction coefficients which also contains the principal diagonal ($\varphi_{ii} \neq 0$).

The mean direction coefficients for i-th and k-th annular zones are determined from the expression for numerical integration written in the final form [1]

$$\varphi_{ik} = \varphi_{F_i - F_k} = 4\frac{N\theta_k}{F_i} \sum_{j1=1}^{N\theta_i} \sum_{i1=1}^{N3_i} \Delta F_{i1} \sum_{k1=1}^{N3_k} \frac{\cos\alpha_{i1}\cos\alpha_{k1}}{\pi r^2} \Delta F_{k1}, \tag{4}$$

where $N\theta_i$, $N\theta_k$ and $N3_i$, $N3_k$ are the numbers of subdivisons of an i-th zone and a k-th zone along angle θ and along the X axis, respectively, for their approximation with polyhedra; r is the distance between the centers of elementary plane areas i1 and k1; ΔF_{i1} and ΔF_{k1} are the areas of elementary areas approximating the annular zones i and k; α_{i1} and α_{k1} are the visibility angles.

On the basis of this algorithm, a program for calculating the mean direction coefficients has been developed in the ALGOL-60 algorithmic language for a computer with BÉSM-ALGOL and TA-1M translators as well as in the FORTRAN-4 algorithmic language for an ES (Unified System) computer. The effectiveness of this program was demonstrated on calculation of the direction coefficients for a system of surfaces formed by two coaxial cylinders of equal lengths. This calculation has yielded the direction coefficients φ_{11} and φ_{21} (subscript 1 referring to the outer cylinder with a 0.5-m radius and subscript 2 referring to the inner cylinder with a 0.15-m radius, both cylinders being 0.1 m long). The number of subdivisions for approximation of two annular zones circumferentially was 158 for the outer cylinder and 48 for the inner cylinder, the number of their subdivisions along the axis being 5. The total number of plane elementary areas in the system was 1030. The machine time for solving this control problem was 14 sec. A comparison of the results φ_{11} = 0.0750 and φ_{21} = 0.1613 with tabulated data [4] indicates a close agreement (error of φ_{ik} determination not larger than 1%).

This algorithm of determining the direction coefficients, which takes into account axial geometrical and opticothermal symmetry of structures tested in radiative heating stands with cylindrical radiators, simultaneously shortens the computer time for calculating φ_{ik} by one order of magnitude and improves the accuracy of these calculations so that a more accurate solution to problems of radiative heat transfer in axisymmetric three-dimensional systems is obtained faster than by solution of analogous problems in the three-dimensional formulation.

Internal Problem of Heat Conduction. The buildup of temperature fields within some two-dimensional region G can be determined from the solution to the two-dimensional heat-conduction equation

$$c\rho \frac{\partial T}{\partial \tau} = \mathrm{div}(\lambda\, \mathrm{grad}\, T) + k\, \mathrm{grad}\, T + q_v \tag{5}$$

for the boundary conditions

$$\tau = 0, \quad T = T_0, \tag{6}$$

at boundary S_1

$$T = T_1, \tag{7}$$

and at boundary S_2

$$-\lambda \frac{\partial T}{\partial n} = \alpha (T_a - T_2) + q_\Sigma, \tag{8}$$

where $q_\Sigma = \varepsilon(q_R - \sigma_0 T_2^4)$. The sum of boundaries S_1 and S_2 constitutes the total boundary of the two-dimensional region. Condition (8) together with the condition of equal temperatures at boundary S_2 for the external problem and for the internal problem will constitute the condition of coupling.

Equation (5) is solved numerically by the method of finite elements, according to the Bubnov–Galerkin procedure. The given region is subdivided into finite elements of triangular shape. On each element of the grid is stipulated a form function N_q (ξ, η), equal to 1 at a q node of the grid and equal to 0 at all other nodes and outside the element. Along with the form function is also introduced a weight function $M_p(\xi, \eta)$ defined on some other grid and having the same properties as function N_q. The nodes of the grids at which functions N_q and M_p are, respectively, stipulated coincide.

The unknown function T in the given region will be expressed in the form

$$\tilde{T} = \sum_{q=1}^{N} N_q T_q, \tag{9}$$

where T_q is the value of $\tilde{T}$ at the q-th node and N is the number of nodes in the grid.

Inserting expression (9) into Eq. (5) yields the discrepancy

$$\omega = \mathrm{div}(\lambda\, \mathrm{grad}\, \tilde{T}) + k\, \mathrm{grad}\, \tilde{T} + q_v - c\rho \frac{\partial \tilde{T}}{\partial \tau}, \tag{10}$$

equal to the difference between the exact solution and the approximate one. In order to make it approach zero, it is necessary to satisfy the condition of orthogonality

$$\int_G M_p \omega\, dG = 0 \tag{11}$$

or

$$\int_G M_p \left[\mathrm{div}(\lambda\, \mathrm{grad}\, \tilde{T}) + k\, \mathrm{grad}\, \tilde{T} + q_v - c\rho \frac{\partial \tilde{T}}{\partial \tau} \right] dG = 0. \tag{12}$$

For the purpose of reducing the order of Eq. (12), we will integrate it by parts. Applying now the Ostrogradskii–Gauss theorem and attaching to Eq. (12) the boundary condition (8), then using relation (9), we obtain

$$\begin{aligned} &\int_G [\mathrm{grad}\, M_p \sum_{q=1}^{N} \lambda\, \mathrm{grad}(N_q T_q) - M_p \sum_{q=1}^{N} k\, \mathrm{grad}(N_q T_q)]\, dG + \\ &+ \int_{S_2} M_p \alpha N_q T_q\, dS - \int_G M_p q_v\, dG + \\ &+ \int_G c\rho M_p \sum_{q=1}^{N} N_q \dot{T}_q\, dG + \int_{S_2} [M_p q_\text{п} - M_p \alpha T_s]\, dS = 0, \end{aligned} \tag{13}$$

where $\dot{T}_q = \partial T_q / \partial \tau$. Equation (13) can be written for each point in region G. The result will be a system of N equations put in matrix form as

$$[P]\{\dot{T}\} + [H]\{T\} + \{F\} = 0. \tag{14}$$

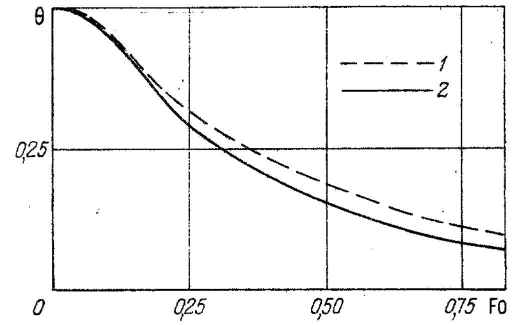

Fig. 3. Comparison of numerical solution (1) and analytical solution [6] (2) to the problem with boundary conditions of the I-III kind, with $N_{Bi} = 0.1$.

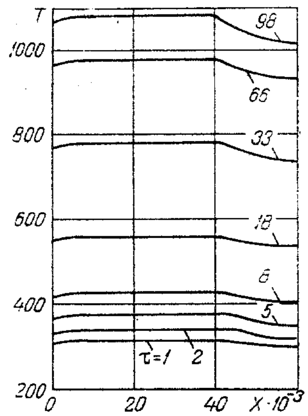

Fig. 4. Change in temperature of heated surface of axisymmetric structure, at density of incident thermal radiation flux $E = 1.26 \cdot 10^6$ W/m² at x = 0.028 m: $T(\overset{\circ}{K})$, X(m), τ(sec).

The coefficients in matrices [P], [H], and {F} will be represented as

$$p_{pq} = \int_G c\rho M_p N_q dG;$$

$$h_{pq} = \int_G [\lambda \operatorname{grad}(N_q) \operatorname{grad}(M_p) - M_p k \operatorname{grad}(N_q) + M_p \alpha N_q] dG;$$

$$f_p = -\int_G M_p q_v dG + \int_{S_2} M_p (q_\Sigma - \alpha T_a) dS; \tag{15}$$

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{Bmatrix} \text{ is the vector-matrix } T_q; \tag{16}$$

$$\{\dot{T}\} = \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \vdots \\ \dot{T}_N \end{Bmatrix} \text{ is the vector-matrix } \dot{T}_q. \tag{17}$$

The system of equations (14) is solved by means of a finite-difference scheme which reduces it to a system of algebraic equations. It must be taken into consideration that system (14) is generally a nonlinear one and to be solved by an iteration process. For the center of the time interval from τ to τ + Δτ one can write

$$\left([H] + \frac{2[P]}{\Delta\tau}\right) \{T\}_{\tau+\Delta\tau/2} = \frac{2[P]}{\Delta\tau} \{T\}_\tau - \{F\}, \tag{18}$$

where [H], [P], and {F} are defined at that center point. The temperatures at the end of such an interval are determined from the relation

$$\{T\}_{\tau+\Delta\tau} = 2\{T\}_{\tau+\Delta\tau/2} - \{T\}_{\tau}. \tag{19}$$

In order to evaluate the integrals in expressions (15), it is necessary to stipulate the form of functions N_q and M_p. From the practical standpoint it is most expedient to stipulate them as linear functions of the coordinates

$$N_q = (a_q + b_q\eta + c_q\xi)/2\Delta, \tag{20}$$

$$M_p = (a_p + b_p\eta + c_p\xi)/2\Delta, \tag{21}$$

where a_q, b_q, ..., c_p are functions of the coordinates of nodal points and Δ is the area of a finite element.

The region for which calculations are made can have any axisymmetric geometry and consists (in general case) of four subregions, each of a different kind and with different physical properties. Those properties can also depend on the temperature and on the direction, which makes it possible to study real objects by taking into account physical nonlinearity and anisotropy of the material. A program had been developed for thus calculating the temperature field of a plate made of a material with the physical characteristics $\rho c = 0.2678 \cdot 10^{-7}$ J/(m^3•K) and $\lambda = 225$ W/(m•K). An examination of the graphs in Figs. 2 and 3 will reveal that the results of numerical calculation agree closely with known analytical solutions [6]. The graph in Fig. 4 depicts the results of heating calculations for an axisymmetric structure whose heated surface is a cylinder (0.03 m in diameter) coupled to a truncated cone (outside diameter of cone 0.068 m). The length of the heated structure is X = 0.06 m in the direction of its axis of rotation. The material of this structure is graphite, its orthotropic thermal conductivity being strongly temperature dependent. The radiative heater is located inside the structure. The results indicate that the temperature of the cylindrical part (X = 0-0.04 m) drops somewhat, owing to partial emission of radiation. The temperature of the conical part (X = 0.04-0.06 m) drops sharply because of emission of radiation as well as because of a decrease in the density of the resultant thermal radiation flux. In the design of a structure one usually knows, with some degree of accuracy, how the thermal fluxes acting on it under actual service conditions will vary in time. By mathematically modeling the heating of that structure in the test stand, one can match the operating modes of radiative heaters so as to simulate the action of the actural thermal fluxes. By analyzing the temperature fields at the same time, one can recommend improvements in the design before performing the control tests.

NOTATION

E, radiation flux density; R, reflection coefficient;, k, kernel of the integral equation; c, specific heat; ρ, density of the material; T, temperature; λ, thermal conductivity; q_V, density of volume heat sources; τ, time; α, heat-transfer coefficient; ε, emissivity; q_R, radiation flux density; σ_0, Stefan–Boltzmann constant. Subscripts: eff, effective value; c, intrinsic value; and a, ambient medium.

LITERATURE CITED

1. E. K. Belonogov and S. N. Shchugarev, "Software for solution of problems of radiative heat transfer during thermal testing of structural elements in radiative heating stands," in: Development of Stands and Methods for High-Temperature Testing [in Russian], Trudy Nauch.-Tekh. Otd., reg. No. 81096149, invent. No. B 983699, Moscow Higher Technical School, Moscow (1980), pp. 72-275.

2. N. N. Chentsov, G. V. Dumkina, and N. S. Shoidina, "Calculation of direction coefficient for radiation in axisymmetric geometry," Inzh.-Fiz. Zh., 34, No. 2, 306-312 (1978).
3. Yu. A. Zhuravlev et al., "Calculation of direction coefficients for radiation in multizonal axisymmetric systems by method of statistical tests," Teplofiz. Vys. Temp., 17, No. 6, 1278-1285 (1979).
4. O. N. Favorskii and Ya. S. Kadaner, Problems of Heat Transfer in Outer Space [in Russian], Vysshaya Shkola, Moscow (1972).
5. L. J. Segerlind, Applied Finite Element Analysis, Wiley (1976).
6. A. I. Pekhovich and V. M. Zhidkikh, Calculation of Thermal State of Solids [in Russian], Énergiya, Leningrad (1976).

MATHEMATICAL MODELING OF HEAT TRANSFER IN SCREENED FURNACES OF RADIAL—CYLINDRICAL AND BOX TYPE

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A zonal model of the heat transfer in screened furnaces is proposed. Theoretical dependences for the distribution of the heat fluxes in the combustion chambers of industrial tube furnaces of two types are given.

Increasing the unit power of furnace equipment requires increase in their size and improvement in the sealing of the structure, which complicates experimental investigations. This has led to an increase in the role of mathematical modeling of the heat transfer in furnaces. Zonal methods are currently the most promising for theoretical calculations.

A distinguishing feature of screened furnaces is the complexity and diversity of their construction and also of the methods of organizing the furnace processes, which is extremely difficult to take into account in theoretical models. Simplified schemes are usually used here.

In the present work, an attempt is made to develop a sufficiently universal zonal method of calculation for screened furnaces. To this end, all chambers are divided into two classes. The first consists of chambers of radial—cylindrical type. These are characterized by the presence of cylindrical, conical, annular, and disk surfaces, bounding both the chamber itself and the isolated zones. The second class comprises furnaces of box type, whose volume is bounded by plane surfaces.

The physical parameters of the medium are assumed to be constant within the limits of each zone and to change discontinuously at the zone boundaries.

To universalize the method, the zones are classified in terms of geometric (Tables 1 and 2) and optical (Tables 3 and 4) features. The plus and minus signs in the tables denote the presence or absence of the corresponding initial parameters for zones of different geometric types.

The geometrical zonal model of a furnace may be considered in the form of a set of zones of various geometric forms included in the classification tables, which allows the constructional features of the particular furnaces to be taken into account.

The calculational system of equations used in the mathematical model to find the zonal mean temperatures and heat fluxes takes the form [1]

$$\sum_{i=1}^{N} P_{ij}T_i^4 + \sum_{\beta=1}^{N}\sum_{\gamma=1}^{N} (\delta_\gamma^j - \delta_\beta^j)\Omega_{\beta\gamma}T_\beta + C_j = 0,\ j = 1,\ 2,\ \ldots,\ N. \quad (1)$$

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